

Superdeformed Oblate Superheavy Nuclei?

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We study stability of superdeformed oblate (SDO) superheavy $Z \geq 120$ nuclei predicted by systematic macroscopic-microscopic calculations in 12D deformation space and confirmed by the Hartree-Fock calculations with the realistic SLy6 force. We include into consideration high- K isomers that very likely form at the SDO shape. Although half-lives $T_{1/2} \lesssim 10^{-5}$ s are calculated or estimated for even-even spin zero systems, decay hindrances known for high- K isomers suggest that some SDO superheavy nuclei may be detectable by the present experimental technique.

The question of what is the largest possible atomic number Z_{max} of an atomic nucleus is still unsettled. The recent experiments on heavy ion fusion in Dubna claim $Z_{max} \geq 118$ [1], with a partial confirmation of hot fusion cross-sections coming from GSI [2] and LBL Berkeley [3]. Predictions on the stability of superheavy nuclei are based either on the Hartree-Fock (HF) studies with some effective interaction chosen out of the existing multitude, or on the more phenomenological, but also more tested, macroscopic-microscopic method. Although these models differ quantitatively, they consistently predict prolate deformed superheavy nuclei with $Z = 100 - 112$, which is confirmed experimentally for nuclei around ^{254}No [4], and spherical or oblate deformed systems with $Z \geq 114$ and $N = 174-184$, see e.g. [5, 6]. In the present letter we show that realistic calculations predict superdeformed oblate (SDO) nuclei, with characteristic quadrupole deformations $-0.4 \lesssim \beta_{20} \lesssim -0.5$ (spheroids with the axis ratio $\approx 3:2$), for $Z \geq 120$. By this we confirm and extend one of the conflicting conclusions of [5] (Fig.12 there). Relying on the calculated energy surfaces, masses and cranking mass parameters, we calculate or estimate half-lives for selected even-even SDO systems. Then we consider an idea, advanced e.g. in [7], of extra stable high- K shape isomers, also in odd systems, whose existence at the SDO shape is very likely. Expected decay hindrances point to the possibility that some of these exotic-shaped superheavy nuclei, far from the conventionally expected "island of stability", live long enough to be detected.

The Model. Within the macroscopic-microscopic method, energy of a deformed nucleus is calculated as a sum of two parts: the macroscopic one being a smooth function of Z , N and deformation, and the fluctuating microscopic one that is based on some phenomenological single-particle (s.p.) potential. A deformed Woods-Saxon potential model used here is defined in terms of the nuclear surface, as exposed in [8]. We admit shapes defined by the following equation of the nuclear surface:

$$R(\theta, \varphi) = c(\{\beta\})R_0\{1 + \sum_{\lambda \geq 1} \beta_{\lambda 0} Y_{\lambda 0}(\theta, \varphi) + \sum_{\lambda \geq 1, \mu > 0, \text{even}} \beta_{\lambda \mu c} Y_{\lambda \mu}^c(\theta, \varphi)\}, \quad (1)$$

where $c(\{\beta\})$ is the volume-fixing factor. The real-valued

spherical harmonics $Y_{\lambda \mu}^c$, with even $\mu > 0$, are defined in terms of the usual ones as: $Y_{\lambda \mu}^c = (Y_{\lambda \mu} + Y_{\lambda -\mu})/\sqrt{2}$. In other words, we consider shapes with two symmetry planes. For the macroscopic part we used the Yukawa plus exponential model [9]. All parameters used in the present work, determining the s.p. potential, the pairing strength and the macroscopic energy, are equal to those used previously in the calculations of masses [10] and fission barriers [11] of heaviest nuclei.

Calculations. We used a rich variety of shapes, with possible nonaxiality and mass-asymmetry, to reliably determine energy landscapes of the heaviest nuclei. A deformation set included both traditional quadrupole deformations β and γ , where $\beta_{20} = \beta \cos \gamma$ and $\beta_{22c} = -\beta \sin \gamma$ (for $\gamma = n \times 60^\circ$, with n integer, a quadrupole shape is axially symmetric), three hexadecapole distortions $\beta_{40}, \beta_{42c}, \beta_{44c}$, the higher-rank even axial multipoles β_{60} and β_{80} , and the following odd-multipole deformations: $\beta_{30}, \beta_{32c}, \beta_{50}, \beta_{52c}$ and β_{70} - altogether twelve parameters. The range of deformation parameters covered a region of shapes up to, and little behind the fission barrier, where the shape parametrization (1) may be hoped sufficient.

Energy landscapes were obtained by a multidimensional energy minimization on a map of equidistant mesh points $(\beta \cos \gamma, \beta \sin \gamma)$ with respect to 10 other deformations. We used a rather large mesh spacing of 0.05 in order to make time-consuming calculations feasible. A subsequent interpolation served to visualize results. In order to check the results we monitored the continuity of the resulting 10 deformation parameters with respect to $\beta \cos \gamma$ and $\beta \sin \gamma$, and their stability with respect to the choice of their starting values. To assess the latter, we repeated the minimization for the whole map for selected nuclei by choosing random starting values. We have found that the results agreed with the ones obtained previously. Additional minimizations have been done to further verify the found minima, in particular, the axially symmetric minima were reproduced by the minimization over the axially symmetric deformations $\beta_{\lambda 0}$.

Equilibria & SDO minima; fission barriers. Quadrupole deformations β_{20} of the g.s. (global) minima, calculated for ~ 300 even-even nuclei are shown in Fig. 1. In addition to spherical, well- or weakly

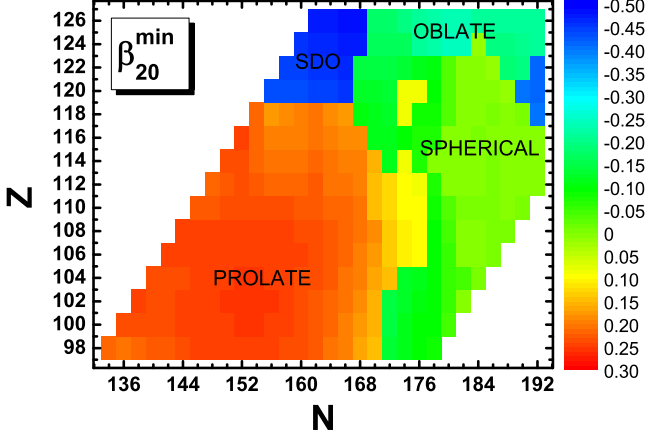


FIG. 1: Calculated ground state quadrupole deformations β_{20} (color online).

deformed prolate and oblate equilibrium shapes there is a region of SDO nuclei for $Z \geq 120$, $N \leq 168$, of particular interest here. SDO global minima occur also for large $N = 190, 192$ and $Z = 118-122$. Although some weakly deformed minima have non-axial distortions, energies of $Z = 120$ isotopes plotted vs. β_{20} for axially symmetric shapes in Fig. 2 fairly illustrate the shape competition & coexistence in the $Z \geq 120$ region. The secondary SDO minima exist there for $168 \leq N \leq 172$ and $N \geq 184$. They appear also in $Z \leq 118$ nuclei. Typically, they lie ≈ 2 MeV above the g.s. This has an effect on the α -decay of the SDO $Z = 120$ isotopes (see below). In the whole $Z \geq 114$ region, the deepest minima, spherical or oblate, occur for $N = 174-184$; for $Z = 124, 126$ they are predominantly oblate.

Energy maps in $(\beta \cos \gamma, \beta \sin \gamma)$ plane are necessary to appreciate fission barriers, Fig 3. The conspicuous result of our calculations is that triaxial saddles occur in all studied nuclei. They may lower the axial fission barrier by up to 2.5 MeV. This lowering increases with N and is larger for bigger Z . The odd-multipole deformations do not change the barriers as much, but they lower some oblate minima and modify the energy maps around and beyond the saddles.

Crucial for stability is that barriers diminish with N decreasing below 174-176 and with Z approaching 126. The first feature is common also to the self-consistent HF results [12], while the second is very distinctive for the macroscopic-microscopic model used here [11]. Hence, the largest barriers of ≈ 3.4 MeV predicted for SDO nuclei $^{286}120$ and $^{288}122$ are rather small as compared to the 5.6 MeV barrier for $^{296}120$ [11]. The barriers for $N \geq 190$ SDO nuclei are still smaller, so we do not consider them further. As the α -decay rates increase with Z , we concentrate on the SDO nuclei around $Z \approx 120$ and $N \approx 166$.

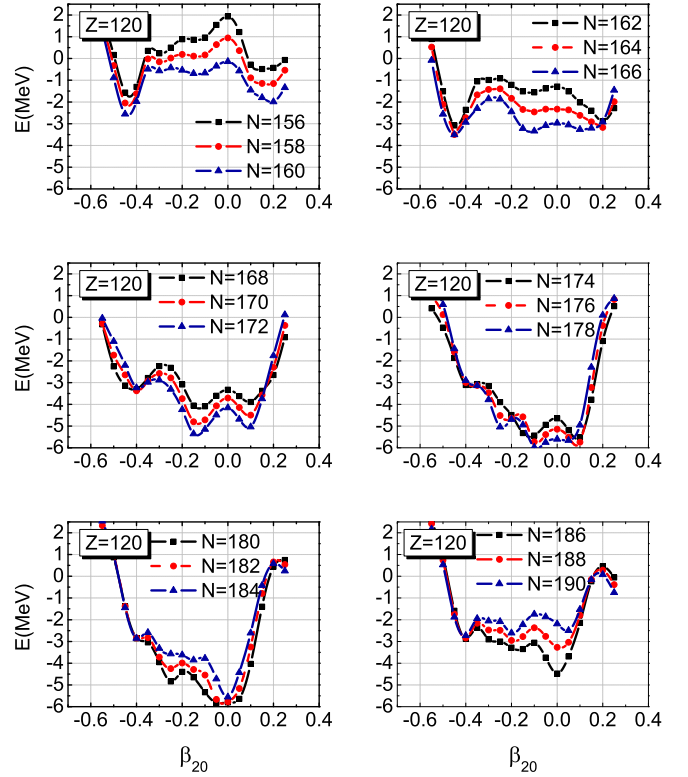


FIG. 2: Energy relative to the spherical macroscopic contribution, $E(\beta_{20}) - E_{macr}(\text{sphere})$, for the $Z = 120$ isotopic chain; each point results from the minimization over $\beta_{\lambda 0}$, $\lambda = 3-8$ (color online).

Other models. To convince ourselves that the SDO minima are not a strange twist of the particular model we repeated the minimizations for the interesting nuclei by using A) the same microscopic model and another version of the macroscopic energy, the LSD liquid drop model of [13], B) the selfconsistent HF method with the realistic Skyrme SLy6 force [14]. Both calculations support the prediction of the global SDO minima; they are even by ≈ 1 MeV deeper with the LSD variant of the macroscopic energy. In the HF calculations, the energy competition between prolate, oblate and SDO minima and fission barriers come out similar as in the macroscopic-microscopic study.

Stability against fission. We checked fission half-lives T_{sf} by calculating WKB action with cranking mass parameters for selected nuclei. We assumed the zero-point energy of 0.5 MeV. To handle fission paths in 12D deformation space we calculate, instead of the mass parameter tensor, the effective mass parameter along a prescribed path. Technically, this is done by replacing analytic derivatives with respect to deformations by the finite differences.

Two possible classes of fission paths and barriers along them may be read from Fig. 3. The barriers along the axial saddle (at $\beta \approx 0.3$, $\gamma = 0$) are longer and have thinner peaks. They can compete with the triaxial path only

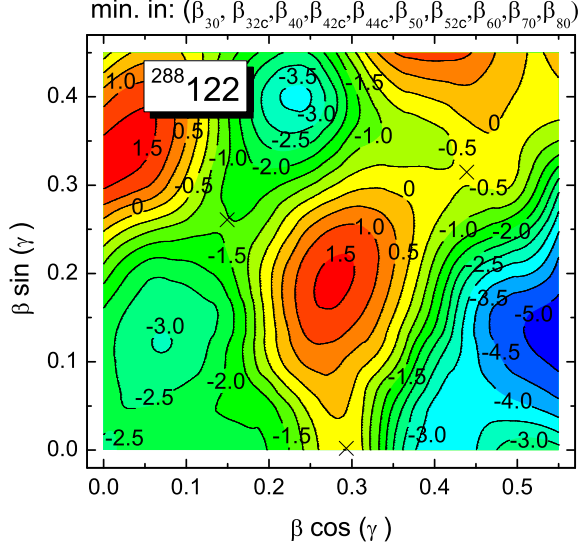


FIG. 3: Energy surface of $^{288}122$, normalized as in Fig.2. Crosses mark the saddles (color online).

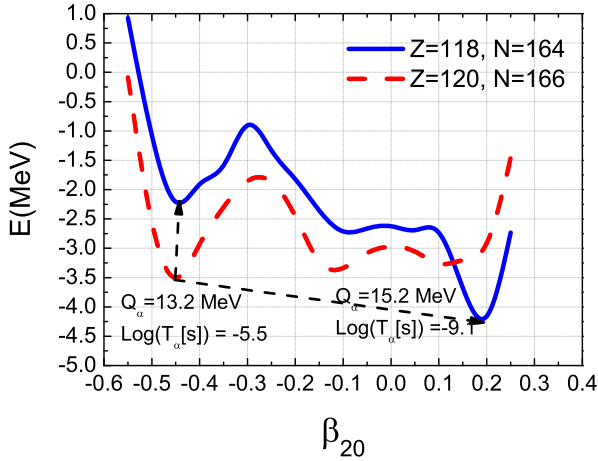


FIG. 4: Mechanism of the α -decay hindrance of the SDO $^{286}120$; energy normalized as in Fig.2 (color online).

when there is a deep normal oblate minimum, i.e. for $N = 166$ or 168 . Triaxial barriers and the related WKB action change smoothly from isotope to isotope. The smallest action we found along triaxial, nearly straight paths. They give half-lives 10^{-6} s for $^{286}120$ and 10^{-5} s for $^{288}122$, with an estimated error of 1 order of magnitude.

Stability against α -decay. From the calculated masses and the improved formula a la Viola-Seaborg [15], we obtain for the g.s.→g.s. transitions $\text{Log}(T_\alpha[\text{s}]) = -9.1$ for $^{286}120$, and *longer* T_α for the lighter $Z = 120$ isotopes. These SDO→prolate transitions, Fig.4, must be strongly

hindered by a very different structure of both configurations, in particular, the occupation of intruder states at SDO shape (see below). If the hindrance would be complete, only SDO→SDO transitions would remain. As already mentioned, SDO configurations in the $Z = 118$ daughters are excited by ≈ 2 MeV (2.5 MeV in HF). This leads to a considerable increase in half-life: $\text{Log}(T_\alpha[\text{s}])$ becomes equal to -5.5 for $^{286}120$ and T_α are *shorter* for lighter isotopes. Clearly, this result holds for the configuration hindrance factors $\leq 10^{-3.6}$.

K-isomers at SDO deformation; odd systems. With half-lives $T_{1/2} < 10^{-5}$ s - the present limit for detection of synthesized superheavy nuclei - superheavy SDO systems might be considered merely as a theoretical curiosity. A fascinating possibility for their longer life-times is related to K - isomerism, see [7, 16]. Indeed, high- K configurations at the SDO shape are very likely, see Fig. 5. Due to large deformation, the neutron $k_{17/2}$ and proton $j_{15/2}$ intruder states with large angular momentum projections on the symmetry axis Ω are close to the Fermi level for $Z = 120$, $N = 166$. Of unique structure and parity, they provide identity to high- K 2(4)-quasiparticle configurations. Candidates for low-lying K -isomers are the so called "optimal" configurations [17], with singly occupied large- Ω orbitals close to the Fermi level. In $^{286}120$, the candidates are the proton $(13/2^-, 7/2^+)10^-$ and neutron $(15/2^+, 9/2^-)12^-$ configurations. The possible low-lying or ground states in odd nuclei are the neutron $15/2^+$ state in $^{285}120$ and the proton $13/2^-$ state in $^{285}119$; the low-lying 14^- state could be expected in the odd-odd $^{284}119$. Detailed predictions would require energy minimization at fixed configuration with blocking.

In assessing stability of high- K isomers or odd nuclei we rely on estimates and analogies with well established experimental facts, as we cannot precisely calculate their decay rates. Let us notice that the considered SDO nuclei are proton-unstable, but in view of the large Coulomb barrier the related life-times may not concern us, at least for even- Z nuclei [5]; odd- Z , high- K states are protected by the centrifugal barrier for high- Ω protons.

Fission hindrance. As well known, T_{sf} for odd and odd-odd heavy and superheavy nuclei are by 3-5 orders longer than for their even-even neighbours. Similar increase was found for high- K isomers, with respect to (prolate) shape isomers on which they are built, in even $^{240-244}\text{Cm}$ [18]. For SDO superheavy K -isomers two factors combine to increase fission half-life: A) the axial fission path is closed by the conservation of the K quantum number, B) triaxial barriers increase due to a decrease in pairing caused by the blocking of two neutrons or protons. Additional hindrance of fission is expected for configurations involving blocked high- Ω intruder states.

Consider now the effect of the saddle deformation on the fission barrier at high spins. The geometrical moments of inertia from the HF calculation in $^{288}120$ are: $\mathcal{J}_\perp = 71$ b, $\mathcal{J}_\parallel = 109$ b at the SDO shape and $\mathcal{J}_\perp^b = 107$ b at the triaxial barrier. The actual moment of inertia at the barrier is reduced by pairing to $f^b \mathcal{J}_\perp^b$, with f^b

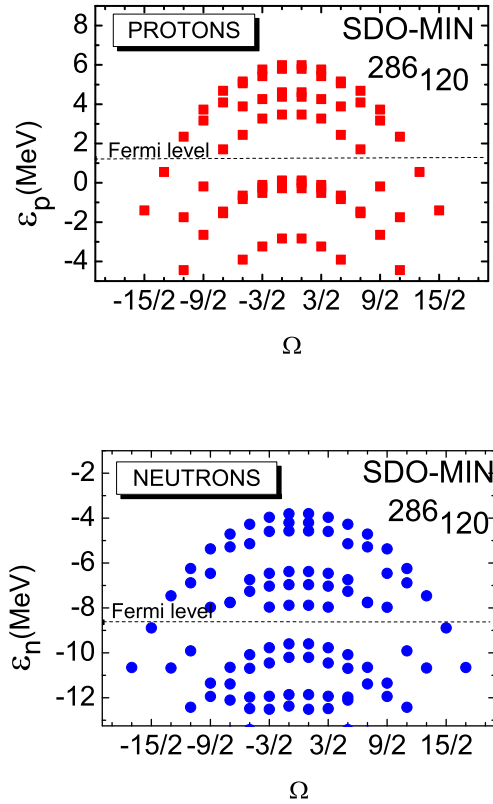


FIG. 5: Calculated s.p. energies vs projection of the angular momentum on the symmetry axis in $^{286}_{120}$ at the SDO g.s.: $\beta_{20} = -0.449$, $\beta_{40} = 0.054$, $\beta_{60} = 0.017$, $\beta_{80} = -0.017$ (color online).

substantially smaller than 1. Without pairing, \mathcal{J}_{\parallel} is an *average* moment of inertia of yrast non-collective high- K states [17]. As $\mathcal{J}_{\parallel} > f^b \mathcal{J}_{\perp}^b$ and pairing at SDO g.s. is *weaker* than at the barrier, there should be *no* decrease

with K in fission barrier for SDO K -isomers.

Alpha-decay hindrance. Although this seems the least certain of our arguments, K -isomerism *may* substantially increase α half-lives: the high- K isomer in ^{270}Ds has longer (partial) half-life $T_{\alpha} = 6.0^{+8.2}_{-2.2}$ ms than the g.s., $T_{\alpha}(\text{g.s.}) = 100^{+140}_{-40}$ μs [19]. For SDO nuclei, an additional hindrance may result from a difference between the parent and daughter high- K configuration, or, for the same configuration, from its extra excitation in the daughter, leading to a smaller Q_{α} .

Stability against beta-decay. The β^+ decay rates λ_{β} for neutron-deficient candidates for the SDO K -isomers can be estimated by neglecting the emitted electron energy $m_e c^2$ in the decay energy: $Q_{\beta} = (M(A, Z) - M(A, Z - 1) - m_e)c^2$. Then, one has $\lambda_{\beta} \sim |M|^2 G_F^2 Q_{\beta}^5$, where $|M|$ is the transition matrix element and G_F is the Fermi constant. Even for a perfect overlap, $|M|^2 \sim 1$, using our calculated masses we obtain half-lives $T_{\beta} = \ln 2 / \lambda_{\beta}$ of the order of 0.1-1 s for even and odd SDO nuclei, consistent with the results by Möller et al. [20]. Since for high- K isomers $|M|$ is reduced, their β^+ decay is even slower.

Although the production of SDO nuclei is another subject, one may notice here that the SDO shape is much closer to the sticking point configuration of the prolate and spherical heavy ions in the side collision than the sphere.

Summarizing, within both macroscopic-microscopic and Skyrme HF methods, one obtains SDO shapes of the ground- or low excited states of superheavy $Z \geq 120$ nuclei. Although even-even, spin zero nuclei decay by a quick $\sim 10^{-5}$ - 10^{-6} s fission or α -decay, longer half-lives are expected for high- K isomers which very likely exist in some even or odd systems. One case of a sizable α -decay hindrance could make such a system detectable by the present technique.

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